1. A
2. C
3. C
4. D
5. A
6. B
7. E
8. A
9. E
10. B
11. D
12. D
13. C
14. C
15. E
16. E
17. D
18. C
19. D
20. B
21. A
22. D
23. C
24. E
25. B
26. E
27. C
28. B
29. D
30. A

1. **A.** $\begin{vmatrix} 8 & 6 & 7 \\ 5 & 3 & 0 \\ 9 & 1 & 8 \end{vmatrix} = -5\begin{vmatrix} 6 & 7 \\ 1 & 8 \end{vmatrix} + 3\begin{vmatrix} 8 & 7 \\ 9 & 8 \end{vmatrix} - 0\begin{vmatrix} 8 & 6 \\ 9 & 1 \end{vmatrix} = -5(48-7) + 3(64-63) =$ $-5(41) + 3(1) = -205 + 3 = -202$

2. **C.** We will find the inverse via Gauss-Jordan elimination. $\left[\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 3 & 3 & 0 & 0 & 0 & 1 & 0 \\ 0 & 4 & 0 & 4 & 0 & 0 & 0 & 1 \end{array}\right] \rightsquigarrow$ $\left[\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & \frac{1}{4} \end{array}\right] \rightsquigarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{2} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 1 & 0 & -\frac{1}{2} & 0 & \frac{1}{4} \end{array}\right]$. The sum of the elements in the third row of $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 4 & 0 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -\frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & \frac{1}{3} & 0 \\ 0 & -\frac{1}{2} & 0 & \frac{1}{4} \end{bmatrix}$ is $0 + \frac{-1}{2} + \frac{1}{3} + 0 = -\frac{1}{6}$.

3. **C.** $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\cdots\begin{bmatrix} 1 & 9 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix} =$ $\begin{bmatrix} 1 & 1+2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\cdots\begin{bmatrix} 1 & 9 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix} =$ $\begin{bmatrix} 1 & 1+2+3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\cdots\begin{bmatrix} 1 & 9 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1+2+3+\cdots+9+10 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 55 \\ 0 & 1 \end{bmatrix}$. $1 + 55 + 0 + 1 = 57$.

4. **D.** $\begin{vmatrix} 6-\lambda & 2 \\ 9 & -1-\lambda \end{vmatrix} = (6-\lambda)(-1-\lambda) - 18 = \lambda^2 - 5\lambda - 24 = (\lambda-8)(\lambda+3) = 0$. $\lambda_1 = 8$ and $\lambda_2 = -3$, so $|\lambda_1 - \lambda_2| = |8 - -3| = 11$.

5. **A.** $\det(A^3) = (\det(A))^3 = \left(\begin{vmatrix} 0 & 5 & 7 \\ 4 & 3 & 1 \\ 0 & 2 & 3 \end{vmatrix}\right)^3 = \left(-4\begin{vmatrix} 5 & 7 \\ 2 & 3 \end{vmatrix}\right)^3 = \left(-4(15-14)\right)^3 =$ $(-4)^3 = -64$.

6. **B.** The system of equations must have been $\begin{cases} 2x + 4y - 3z = -9 \\ 3x - 2y - 4z = 4 \\ 5x + 8y - z = 1 \end{cases}$. The solution to this system has $x = \dfrac{\begin{vmatrix} -9 & 4 & -3 \\ 4 & -2 & -4 \\ 1 & 8 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & 4 & -3 \\ 3 & -2 & -4 \\ 5 & 8 & -1 \end{vmatrix}} = 4$ and $z = \dfrac{\begin{vmatrix} 2 & 4 & -9 \\ 3 & -2 & 4 \\ 5 & 8 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & 4 & -3 \\ 3 & -2 & -4 \\ 5 & 8 & -1 \end{vmatrix}} = 3$. $x + z = 4 + 3 = 7$.

7. **E**. $M^{10} - M^9 - M^8 = M^8\left(M^2 - M - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$ $= M^8\left(\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = M^8\left(\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) =$ $M^8\left(\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

8. **A.** Multiplying the first row by 2 has the effect of multiplying the determinant by 2. Swapping the second and third row has the effect of negating the determinant. Adding a multiple of one row to another has no effect on the determinant.
9. **E.** The adjoint matrix is the transpose of the matrix of cofactors, so we're really looking for the first entry in the second row of the cofactor matrix. This will be $-\begin{vmatrix} 2 & -1 \\ -3 & 7 \end{vmatrix} = -(14-3) = -11$
10. **B.** $M = PAP^{-1}$ can be rearranged to yield $A = P^{-1}MP = \begin{bmatrix} -2 & -5 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 14 & 30 \\ -6 & -13 \end{bmatrix}\begin{bmatrix} 2 & 5 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} -2 & -5 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} -2 & 10 \\ 1 & -4 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$.
11. **D.** $M^{10} = (PAP^{-1})^{10} = (PAP^{-1})(PAP^{-1})(PAP^{-1}) \dots (PAP^{-1}) =$ $PA(P^{-1}P)A(P^{-1}P)A(P^{-1} \dots P)AP^{-1} = PA^{10}P^{-1} = \begin{bmatrix} 2 & 5 \\ -1 & -2 \end{bmatrix}\left(\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}\right)^{10}\begin{bmatrix} -2 & -5 \\ 1 & 2 \end{bmatrix} =$ $\begin{bmatrix} 2 & 5 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1024 \end{bmatrix}\begin{bmatrix} -2 & -5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} -2 & -5 \\ 1024 & 2048 \end{bmatrix} = \begin{bmatrix} 5116 & 10230 \\ -2046 & -4091 \end{bmatrix}$.
12. **D.** $\sum_{k=1}^{3}\begin{bmatrix} 1 & k & k^2 \\ k & k^2 & k^3 \\ k^2 & k^3 & k^4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 8 \\ 4 & 8 & 16 \end{bmatrix} + \begin{bmatrix} 1 & 3 & 9 \\ 3 & 9 & 27 \\ 9 & 27 & 81 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 14 \\ 6 & 14 & 36 \\ 14 & 36 & 98 \end{bmatrix}$.
13. **C**. An idempotent is equal to its own square. $\begin{bmatrix} 2 & -2 & -4 \\ -1 & x & 4 \\ 1 & -2 & -3 \end{bmatrix} =$ $\begin{bmatrix} 2 & -2 & -4 \\ -1 & x & 4 \\ 1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 2 & -2 & -4 \\ -1 & x & 4 \\ 1 & -2 & -3 \end{bmatrix} = \begin{bmatrix} 2 & 4-2x & -4 \\ 2-x & -6+x^2 & -8+4x \\ 1 & 4-2x & -3 \end{bmatrix}$. To make all the entries in the left side of the equality equal to all the entries in the right side of the equality, it is necessary that $x = 3$.
14. **C**. Let the matrix M represent the original message.
    We know $\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} M = \begin{bmatrix} 47 & 42 & 101 & 20 \\ 29 & 28 & 61 & 12 \end{bmatrix}$. Multiplying both sides by $\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix}$ yields $M = \begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 47 & 42 & 101 & 20 \\ 29 & 28 & 61 & 12 \end{bmatrix} = \begin{bmatrix} 11 & 14 & 21 & 4 \\ 7 & 0 & 19 & 4 \end{bmatrix} = \begin{bmatrix} L & O & V & E \\ H & A & T & E \end{bmatrix}$.
15. **E**. With only three cases, it's not too time consuming to just check each choice.
    $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 1 & 3 & 1 \end{bmatrix}\begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix}$. This vector is an eigenvector with eigenvalue 1.
    $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 1 & 3 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. This vector is an eigenvector with eigenvalue 0.
    $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -2 \end{bmatrix}$. This vector is not an eigenvector.
16. **E.** $1\begin{vmatrix} 2 & 0 & 0 \\ 3 & 3 & 0 \\ 4 & 0 & 4 \end{vmatrix} - 1\begin{vmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{vmatrix} + 0\begin{vmatrix} 0 & 2 & 0 \\ 0 & 3 & 0 \\ 0 & 4 & 4 \end{vmatrix} - 0\begin{vmatrix} 0 & 2 & 0 \\ 0 & 3 & 3 \\ 0 & 4 & 0 \end{vmatrix} = \begin{vmatrix} 2 & 0 & 0 \\ 3 & 3 & 0 \\ 4 & 0 & 4 \end{vmatrix} - 0 + 0 - 0 =$ $2\begin{vmatrix} 3 & 0 \\ 0 & 4 \end{vmatrix} - 0\begin{vmatrix} 3 & 0 \\ 4 & 4 \end{vmatrix} + 0\begin{vmatrix} 3 & 3 \\ 4 & 0 \end{vmatrix} = 24 - 0 + 0 = 24$.

17. **D**. Subtracting the second equation from the first yields $w-5z=-21$ which rearranges to $w=-21+5z$. The second equation rearranges to $x=23-3z$. The third equation rearranges to $y=-20+4z$. This means that all solutions $\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix}$ will be in the form $\begin{bmatrix} -21+5z \\ 23-3z \\ -20+4z \\ z \end{bmatrix}$ which may alternatively be written in the form $\begin{bmatrix} -21 \\ 23 \\ -20 \\ 0 \end{bmatrix} + z\begin{bmatrix} 5 \\ -3 \\ 4 \\ 1 \end{bmatrix}$.

18. **C.** $\begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}^6 = \left(\begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}^3\right)^2 = \left(\begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}\right)^2 =$ $\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. The sum of the elements is $1+0+0+1=2$.

19. **D**. The system of equations governing this problem can be represented as $\begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} C \\ B \end{bmatrix} = \begin{bmatrix} 21 \\ 22 \end{bmatrix}$. To solve for $\begin{bmatrix} C \\ B \end{bmatrix}$ we multiply both sides by $\begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}^{-1} = \frac{1}{8}\begin{bmatrix} 4 & -2 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{8} \end{bmatrix}$. We see $\begin{bmatrix} C \\ B \end{bmatrix} =$ $\begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{8} \end{bmatrix}\begin{bmatrix} 21 \\ 22 \end{bmatrix}$.

20. **B.** For the sake of explaining the solution clearly, we will make up variable names for the number of batches of brownies in each scenario, but these are irrelevant to the problem. To solve each of the scenarios, one will, based on the results of the previous problem do:

$$\begin{bmatrix} W \\ J \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{8} \end{bmatrix}\begin{bmatrix} 14 \\ 20 \end{bmatrix} \quad \begin{bmatrix} X \\ K \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{8} \end{bmatrix}\begin{bmatrix} 25 \\ 30 \end{bmatrix} \quad \begin{bmatrix} Y \\ L \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{8} \end{bmatrix}\begin{bmatrix} 11 \\ 18 \end{bmatrix} \quad \begin{bmatrix} Z \\ M \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{8} \end{bmatrix}\begin{bmatrix} 18 \\ 20 \end{bmatrix}.$$

Rather than compute all of these, one should sum them all.

$$\begin{bmatrix} W+X+Y+Z \\ J+K+L+M \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{8} \end{bmatrix}\begin{bmatrix} 14 \\ 20 \end{bmatrix} + \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{8} \end{bmatrix}\begin{bmatrix} 25 \\ 30 \end{bmatrix} + \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{8} \end{bmatrix}\begin{bmatrix} 11 \\ 18 \end{bmatrix} + \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{8} \end{bmatrix}\begin{bmatrix} 18 \\ 20 \end{bmatrix}$$

$$\begin{bmatrix} W+X+Y+Z \\ J+K+L+M \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{8} \end{bmatrix}\left(\begin{bmatrix} 14 \\ 20 \end{bmatrix} + \begin{bmatrix} 25 \\ 30 \end{bmatrix} + \begin{bmatrix} 11 \\ 18 \end{bmatrix} + \begin{bmatrix} 18 \\ 20 \end{bmatrix}\right)$$

$$\begin{bmatrix} W+X+Y+Z \\ J+K+L+M \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{8} \end{bmatrix}\begin{bmatrix} 68 \\ 88 \end{bmatrix} = \begin{bmatrix} 34-22 \\ -17+33 \end{bmatrix} = \begin{bmatrix} 12 \\ 16 \end{bmatrix}$$

21. **A.** $(QRS)^{-1} = S^{-1}R^{-1}Q^{-1} = \begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 2 & -5 \\ 3 & -8 \end{bmatrix}\begin{bmatrix} 4 & -7 \\ -1 & 3 \end{bmatrix} =$ $\begin{bmatrix} -1 & 3 \\ -5 & 14 \end{bmatrix}\begin{bmatrix} 4 & -7 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} -7 & 16 \\ -34 & 77 \end{bmatrix}$

22. **D.** These vectors are linearly dependent if and only if $\begin{bmatrix} x & 7 & 3 \\ -1 & x & x \\ 1 & 5 & 2 \end{bmatrix}$ is singular. We need to find the x value satisfying $0 = \begin{vmatrix} x & 7 & 3 \\ -1 & x & x \\ 1 & 5 & 2 \end{vmatrix}$.
$0 = x\begin{vmatrix} x & x \\ 5 & 2 \end{vmatrix} - 7\begin{vmatrix} -1 & x \\ 1 & 2 \end{vmatrix} + 3\begin{vmatrix} -1 & x \\ 1 & 5 \end{vmatrix} = x(-3x) - 7(-2-x) + 3(-5-x)$
$0 = -3x^2 + 14 + 7x - 15 - 3x = -3x^2 + 4x - 1 = -(3x-1)(x-1)$.
The sum of the solutions is $\frac{1}{3} + 1 = \frac{4}{3}$
23. **C.** The transformation of the human population on a given night H_n and tortoise population on that night T_n to the human population H_{n+1} and tortoise population T_{n+1} on the next night can be represented as $\begin{bmatrix} \frac{3}{5} & \frac{1}{3} \\ \frac{2}{5} & \frac{2}{3} \end{bmatrix}\begin{bmatrix} H_n \\ T_n \end{bmatrix} = \begin{bmatrix} H_{n+1} \\ T_{n+1} \end{bmatrix}$. Since H and T represent equilibrium populations, we know $\begin{bmatrix} \frac{3}{5} & \frac{1}{3} \\ \frac{2}{5} & \frac{2}{3} \end{bmatrix}\begin{bmatrix} H \\ T \end{bmatrix} = \begin{bmatrix} H \\ T \end{bmatrix}$. Because $\frac{3}{5}H + \frac{1}{3}T = H$ we realize $\frac{1}{3}T = \frac{2}{5}H$ which can be manipulated to yield $\frac{H}{T} = \frac{1/3}{2/5} = \frac{5}{6}$. $5 + 2(6) = 17$
24. **E.** $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$. $\begin{bmatrix} 5 & 6 \\ 3 & 4 \end{bmatrix}^{-1} = \frac{1}{20-18}\begin{bmatrix} 4 & -6 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -\frac{3}{2} & \frac{5}{2} \end{bmatrix}$.
25. **B**. $\begin{vmatrix} 5 & 6 \\ 6 & 7 \end{vmatrix} = 5 \times 7 - 6 \times 6 = 35 - 36 = -1$
26. **E.** The first and last row of this matrix are identical. When one row of a matrix is a multiple of another row, the determinant is always zero.
27. **C.** Matrix A, the transpose of the cofactor matrix, is by definition the adjugate matrix of M (i.e. the classical adjoint). Dividing the adjugate of M by the determinant of M is one method of finding the inverse of M. So, $\frac{1}{\det(M)}MA = M\left(\frac{1}{\det(A)}A\right) = M(M^{-1}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. The sum of the entries is $1 + 1 + 1 = 3$.
28. **B.** $A\begin{bmatrix} 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$ and $A\begin{bmatrix} -1 \\ 8 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$ imply $A\begin{bmatrix} -1 & 3 \\ 8 & 6 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$. In an effort to make the right side of the equality the identity matrix, we may rewrite this as $A\begin{bmatrix} -1/4 & 3/4 \\ 2 & 3/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Matrix inverses are unique, so it must be the case that $A^{-1} = \begin{bmatrix} -1/4 & 3/4 \\ 2 & 3/2 \end{bmatrix}$. The sum of the entries is $-\frac{1}{4} + \frac{3}{4} + 2 + \frac{3}{2} = 4$.

29. **D.** $(A+B)^2 = A^2 + 2AB + B^2 \Leftrightarrow A^2 + AB + BA + B^2 = A^2 + 2AB + B^2 \Leftrightarrow AB + BA = 2AB \Leftrightarrow BA = AB \Leftrightarrow A$ and B commute. The first statement is true.

Consider the fact that for any matrix M, $\operatorname{adj} M = \det(M) \cdot M^{-1}$. So, $\operatorname{adj}(AB) = \det(AB) \cdot (AB)^{-1} = \det(A) \cdot \det(B) \cdot B^{-1} \cdot A^{-1} = \det(A) \cdot A^{-1} \cdot \det(B) \cdot B^{-1} = \operatorname{adj}(A) \cdot \operatorname{adj}(B)$. The second statement is true.

Given a nilpotent matix M, there exists a positive integer n such that $M^n = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}$. So, $\det(M^n) = \det\left(\begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}\right)$. We see $(\det(M))^n = 0$ which means $\det(M) = 0$. We conclude that M is singular. The third statement is true.

30. **A.** There is no special trick to this problem. $\begin{bmatrix} 1 & 6 & -4 \\ 7 & 2 & 3 \\ 2 & -5 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 3 \\ -6 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2-24+24 & -1+30-4 & 3+18-4 \\ 14-8-18 & -7+10+3 & 21+6+3 \\ 4+20-6 & -2-25+1 & 6-15+1 \end{bmatrix} = \begin{bmatrix} 2 & 25 & 17 \\ -12 & 6 & 30 \\ 18 & -26 & -8 \end{bmatrix}$